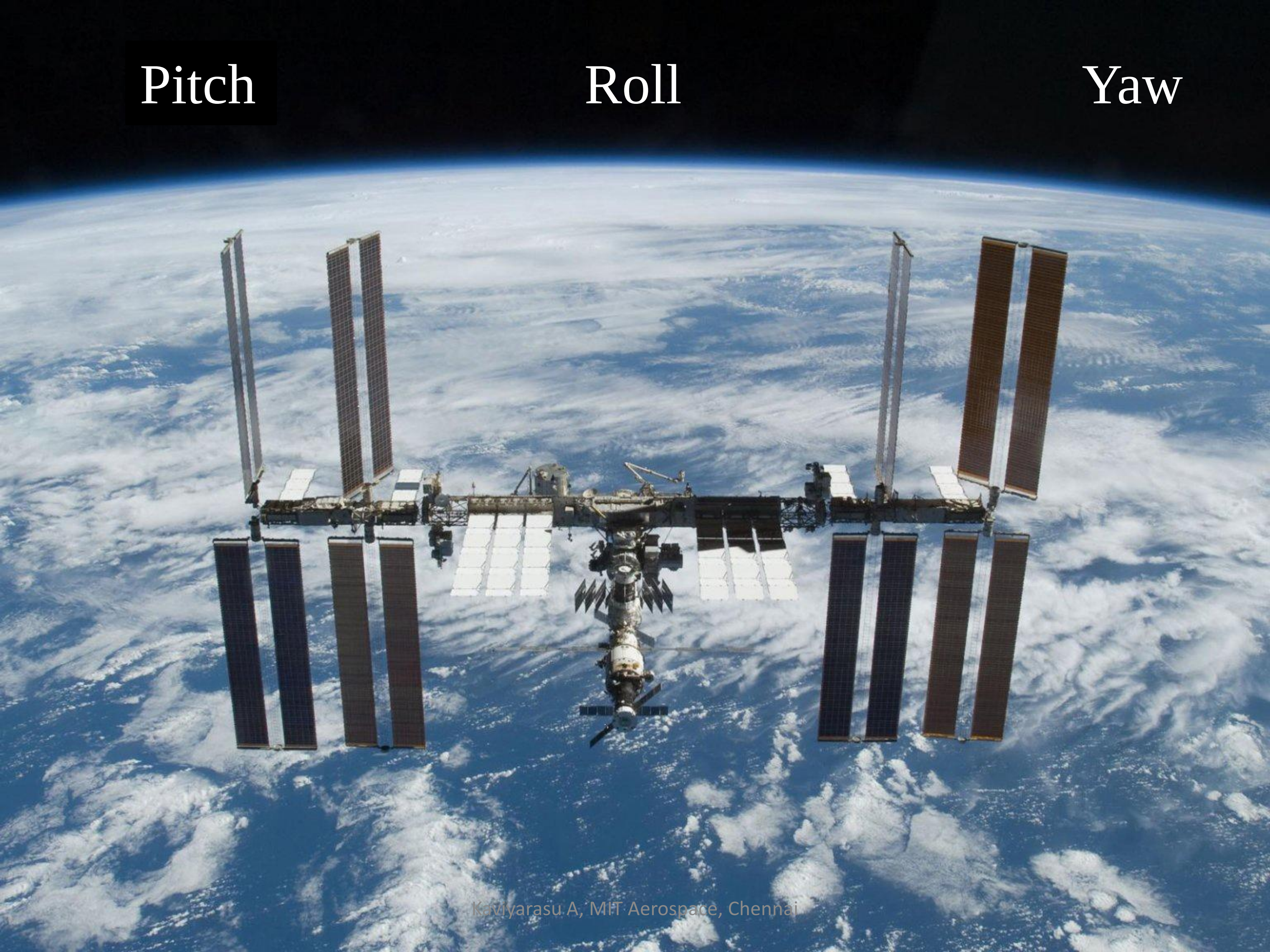
FUNDAMENTALS OF ATTITUDE ESTIMATION

Prepared by

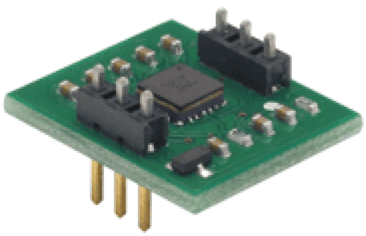
A.Kaviyarasu
Assistant Professor
Department of Aerospace Engineering
Madras Institute Of Technology
Chromepet, Chennai

- Basically an IMU can used for two main purposes
  - To measure aircraft attitude information (Pitch, Roll and yaw)
    - Accelerometers to measure pith and roll information
    - Gyroscope to measure pitch, roll and yaw information
    - Magnetometer to measure pitch and roll information
  - To measure aircraft position information (Dead Reckoning)
    - Stable platform (Gyros and Accelerometer are mounted on a gimbal platform)
    - Strap down platform (Gyros and Accelerometer are Strapped it on body of an aircraft)

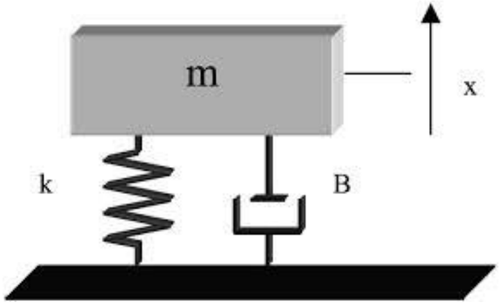

Pitch
Roll
Yaw

Estimation of Angle with Accelerometer

- Three-axis accelerometer can be modeled as



$$a_m = \frac{1}{m}\left(F - F_g\right)$$



a_m = measured acceleration

m = mass of the body

F = sum of all forces acts on body expressed in bodyframe (includesgravity)

F_g = Force due to gravity expressed in bodyframe we assume that F=0 then the output of accelerometer is

$$a_m = \frac{-F_g}{m}$$

In inertial frame force of gravity is expressed as $\begin{pmatrix} 0 \\ 0 \\ mg \end{pmatrix}$

Rotating the gravity into the body frame of the sensor

$$F_g = R_I^B \begin{pmatrix} 0 \\ 0 \\ mg \end{pmatrix} = \begin{pmatrix} -mg\sin(\theta) \\ mg\cos(\theta)\sin(\phi) \\ mg\cos(\phi)\cos(\theta) \end{pmatrix}$$

- The expected accelerometer data in the body frame is defined by

$$a_m = \begin{pmatrix} g\sin(\theta) \\ -g\cos(\theta)\sin(\phi) \\ -g\cos(\phi)\cos(\theta) \end{pmatrix}$$

- Let the component of acceleration is defined by its axis

$$\theta_{accel} = \arcsin\left(\frac{a_{mx}}{g}\right)$$

$$\phi_{accel} = \arctan\left(\frac{a_{my}}{a_{mz}}\right)$$

- The measured angles are only the estimated angles not the actual angels.
- This method provides quick and very simple way of estimating pitch and roll angle from the accelerometer.
- However we assumed that the forces acting on the accelerometer was only gravity.
- Vibration and other external forces will affect our pitch and roll angle measurement.

Estimation of Angle with Gyroscope

- Rate gyro can also be used to measure the attitude of vehicle. unlike accelerometer rate gyros are not affect by external acceleration and external forces. Gyro based measurement is immune to external forces.
- Since roll, pitch, and yaw each occur in different reference frames, we need to take the rate gyro outputs and rotate them into the appropriate frames in order to get the Euler's rate.

$$\begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = \begin{pmatrix} p + q\sin(\phi)\tan(\theta) + r\cos(\phi)\tan(\theta) \\ q\cos(\phi) - r\sin(\phi) \\ q\dfrac{\sin(\phi)}{\cos(\theta)} + r\dfrac{\cos(\phi)}{\cos(\theta)} \end{pmatrix}$$

$$\phi^{+}_{gyro} = \phi^{+}_{gyro} + (dt * \dot{\phi})$$

$$\theta^{+}_{gyro} = \theta^{+}_{gyro} + (dt * \dot{\theta})$$

$$\psi^{+}_{gyro} = \psi^{+}_{gyro} + (dt * \dot{\psi})$$

This method helps to use rate gyros to estimate angles that are **not sensitive to vibration and other external forces**. However rate gyros are noisy and imperfect, every time we add new gyro measurements, we add errors to our angle estimates. Over time, errors will accumulate, causing our gyro-based angles estimates to drift over time.

COMBINING ACCELEROMETER AND RATE GYRO DATA

- Angle estimate based on rate gyros alone drift over time, making them unreliable in the long-term purpose or application.
- Angle estimates based on accelerometers do not cause angle estimates to drift, but they are sensitive to external forces like vibration, making short-term estimates unreliable.
- Here we are going to discuss about how to combine the outputs of both types of sensor to produce angle estimates that are resistant to both vibration and immune to long-term drift.
- Here the step is classified into two types
  - Prediction (by using gyroscope reading to measure incremental changes in the angle).

- Estimation (by means of using accelerometer reading to correct the rate gyro drift).
- In updating step we assume accelerometer based angle estimation is close to true value .
- We take the measured angle (from the accelerometers) and the predicted angle (from the rate gyros), compute the difference, and add a portion of the difference to the final angle estimate.

$$\theta_{estimate} = \theta_{gyro}^{+} + L\left(\theta_{accel} - \theta_{gyro}^{+}\right)$$

$$\phi_{estimate} = \phi_{gyro}^{+} + L\left(\phi_{accel} - \phi_{gyro}^{+}\right)$$

$$\psi_{estimate} = \psi_{gyro}^{+} + L\left(\psi_{mag} - \psi_{gyro}^{+}\right)$$

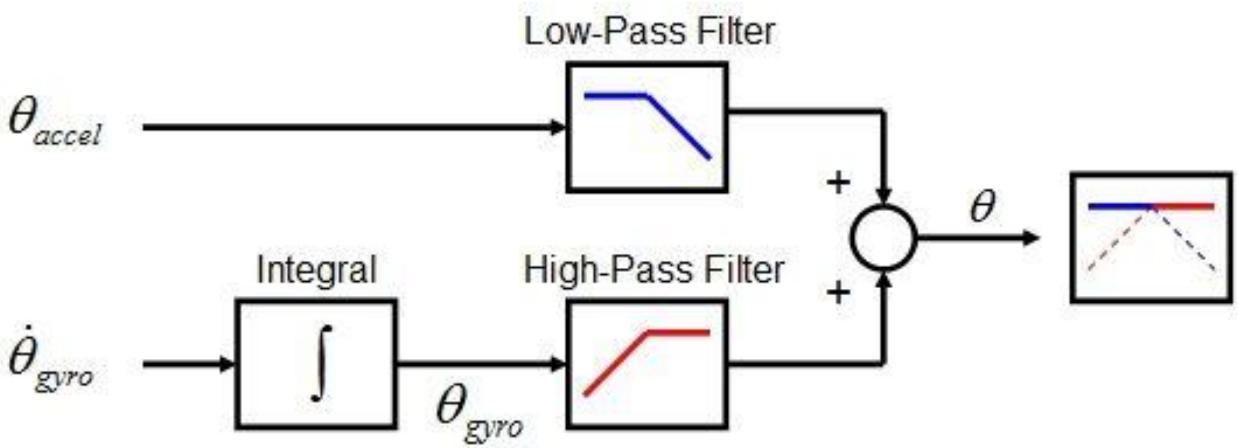
- L is numeric value between 0 and 1.

$$if\;\; L = 0 \rightarrow \text{ gyro based angle mesurement is used}$$
$$\text{(accelerometer measurment become zero)}$$
$$if\;\; L = 1 \rightarrow \text{ accelerometer based angle mesurement is used}$$
$$\text{(gyros measurment become zero)}$$

- As approaches L=1, the accelerometers are trusted more and the rate gyros are trusted less. This makes the angle estimates less sensitive to rate gyro noise and biases, but more sensitive to vibration and other external forces.
- As approaches L=0, rate gyros are trusted more and the accelerometers are trusted less. This makes the angle estimates more sensitive to rate gyro noise and biases, but less sensitive to vibration and other external forces.

- This formulation can be called a “fixed-gain observer” and it looks similar to that of Complementry Filter and Kalman Filter.
- The main difference is that in a Kalman Filter, the observer gain is selected optimally using known characteristics of the physical system.
- In addition, a Kalman Filter can exploit knowledge of the physical system so that accelerometer data (and other data) needn’t be converted to angles before using it to make corrections to the angle estimates.

Complementary Filter



$$angle = 0.98*(angle + gyrodata*dt) + 0.02*(accData);$$

```
// Turning around the X axis results in a vector on the Y-axis
pitchAcc = atan2f((float)accData[1], (float)accData[2]) * 180 / M_PI;
*pitch = *pitch * 0.98 + pitchAcc * 0.02;

// Turning around the Y axis results in a vector on the X-axis
rollAcc = atan2f((float)accData[0], (float)accData[2]) * 180 / M_PI;
*roll = *roll * 0.98 + rollAcc * 0.02;
```

Is accelerometer helps to measure velocity and position?

- The answer is yes…… and no……….
- It depends upon how much accuracy we are in needed for our application.
- The low cost accelerometer are very less in accuracy result in every poor estimation in position and velocity.
- The accuracy not only because of low cost sensor but also for misalignment of sensor position in the body frame. Small misalignment errors may leads to high errors in acceleration measurement, which translate into more error in velocity estimation and much more error in position estimate.

Estimation of Position and Velocity

$$V_i = \int a_i$$

$$P_i = \int\int a_i$$

$V_i \rightarrow inertial\ frame\ Velocity$

$P_i \rightarrow inertial\ frame\ Position$

$a_i \rightarrow Inertial\ frame\ acceleration$

But in practical

$$V_i = V_i + (dt * a_i)$$

$$V_{i+} = dt * a_i$$

$$P_i = P_i + (dt * V_i)$$

$$P_{i+} = dt * V_i$$

Expected Accuracy of Velocity and Position Estimates

Angle Error (degrees)	Acceleration Error (m/s/s)	Velocity Error (m/s) @ 10 seconds	Position Error (m) @ 10 seconds	Position Error (m) @ 1 minute	Position Error (m) @ 10 minutes	Position Error (m) @ 1 hour
0.1	1.71	0.2	1.7	62	6157	221.6 E 3
0.5	8.55	0.9	8.6	308	30.8 E 3	1.1 E 6
1.0	17.11	1.7	17.1	615	61.6 E 3	2.2 E 6
1.5	25.66	2.6	25.6	924	92.4 E 3	3.3 E 6
2.0	34.22	3.4	34.2	1232	123.2 E 3	4.4 E 6

HOW TO REDUCE THE ERROR?

- Reduce the Distortions caused by an Accelerometer.
- Reduce the Distortions caused by gyroscope.
- Reduce the Distortions caused by Magnetometer.
- By using any sensor fusion technique. (Eg.GPS INS Integration).

Distortions by Accelerometer

- The simplified accelerometer mentioned in the above equation will not consider the account of cross-axis misalignment, temperature varying output bias and scale factors. All these factor will affects accelerometer sensor output and affect accuracy of velocity and position measurement.

- The more complete model of the accelerometer is given below

$$a_m = M_a\left(\frac{1}{m} S_a(T)(\mathrm{F} - \mathrm{F_g}) - \beta(\mathrm{t})\right)$$

M_a = acceleration misalignment matrix

$\beta(\mathrm{t})$ = Vector of temperature varying bias

$S_a(T)$ = Diagonal temperature varying accelerometer Sensitivity matrix

$$S_a(T) = \begin{pmatrix} S_{ax} & 0 & 0 \\ 0 & S_{ay} & 0 \\ 0 & 0 & S_{az} \end{pmatrix}$$

- The sensitivity matrix encodes expected accelerometer raw output for a given measured acceleration.
- The misalignment matrix describes the effect of cross-axis misalignment unlike bias and sensitivity terms, it is not affected be temperature.

- When using accelerometer data, we measure only value, is a_m but what we really needed is the actual acceleration a_{mc} before scale factors, biases, and misalignment distort the measurement. That is, we want to take the measurement and extract a_{mc} the term . Solving, we get

$$a_{mc} = S_a^{-1}(T)\left(M_a^{-1} a_m + \beta_a(T)\right)$$

$a_{mc} \rightarrow$ corrected accelerometer measurement vector

In order to obtain the best accuracy, the terms should be determined experimentally over the full operating temperature of the sensor

DISTORTIONS BY GYROSCOPE

- Let the Vector P is defined as $$p = \begin{pmatrix} p \\ q \\ r \end{pmatrix}$$

$$P = S_g^{-1}(T)\left(M_g^{-1} p_m + \beta_g(T) - C_{ga} * a_{mc}\right)$$

$p_m \rightarrow$ measured angular rate vector

$C_{ga} \rightarrow$ matrix encoding the sensitivity of rate gyro axes to acceleration

$a_{mc} \rightarrow$ corrected accelerometer measurement

- Usually coupling between the acceleration and rate gyro output C_{ga} is small enough that can be neglected, but in high acceleration is expected (e.g. on a rocket), this term must be measured and included in the model for the best accuracy.

DISTORTIONS BY MAGNETOMETER

- Magnetometer are very trick one to calibrate.
- Like accelerometer and gyroscope magnetometer also subject to cross-axis misalignment, output bias ,scale factors.
- In addition to that, local magnetic field around the magnetometer can be distorted by magnetic and ferrous metal objects. This distortion must be measured and corrected in order for the magnetometer to be useful.
- The field can also be distorted by time-varying electromagnetic waves from high-power electrical lines and communication lines placed near the sensor.
- Finally, as the sensor is moved around in practice, the field can be distorted so that, in certain locations (say, next to a concrete wall filled with rebar), the accuracy of the measurement is reduced.

$$\mathrm{b}=S_b^{-1}(T)\left(M_b^{-1}b_m+\beta_b(T)\right)$$

$b \rightarrow$ magnetic field

$b_m \rightarrow$ Measured magnetic field

$S_b^{-1} \rightarrow$ inverse of the magnetometer sensitive matrix

$M_b^{-1} \rightarrow$ inverse of the magnetometer misalignment matrix

$\beta_b(T) \rightarrow$ magnetometer bias vector

- After the magnetometer measurement is corrected for axis misalignment, scale factor, and bias errors, it must be corrected for additional local magnetic field distortions.

- Two types of local distortions are there
  - Soft-Iron Distortion.
  - Hard -Iron Distortion.
- A soft-iron field distortion is caused by ferrous metal objects that bend the Earth's magnetic field e.g. Screws
- A hard-iron distortion is caused by near by object having its own magnetic field like permanent magnets motor or current carrying conductors.
- In the absence of any hard or soft-iron distortions, the outputs of the magnetometer as it is rotated should trace out a perfect sphere centered around zero.
- Soft-iron distortions distort the sphere so that it appears to be an ellipsoid. Hard-iron distortions shift the mean of the sphere away from zero

- One way to calibrate the magnetometer for hard and soft iron distortions is to fit an ellipsoid to a set of magnetometer data collected over a wide variety of different orientations. The center of the ellipsoid is equivalent to the bias caused by hard iron distortions, while the ellipsoid shape on the x, y, and z axes are caused by the soft-iron distortions. A correction matrix can then be calculated that, when multiplied by the magnetometer measurements, alters the ellipsoid so that it looks like a sphere again.

SENSOR DATA FUSION

- The sensor fusion can be implemented by some of the steps below

  -Complementary Filter (CF)

  -Kalman Filter (KF)

  -Extended Kalman Filter (EKF)

  -Unscented Kalman Filter (UKF)

  -Particle Filter (PF)

- Nothing but fusion of data’s from two or more sensor.
- The data measured from a single sensor not gives accurate measurement of reading. It may suffer from **some noise ,errors ,misalignment and external disturbance.**
- In order to measure the exact physical quantity of data ,two or more sensors are used. these data's are fused in such a way it will provide original measurement of the physical quantity.

COMBINING ACCELEROMETER AND RATE GYRO DATA

- Angle estimates based on rate gyros alone drift over time, making them unreliable in the long-term purpose or application.
- Angle estimates based on accelerometers do not cause angle estimates to drift, but they are sensitive to external forces like vibration, making short-term estimates unreliable.
- Here we are going to discuss about how to combine the outputs of both types of sensor to produce angle estimates that are resistant to both vibration and immune to long-term drift.
- Here the step is classified into two types
  - Prediction (by using gyroscope reading to measure incremental changes in the angle).

- Estimation (by means of using accelerometer reading to correct the rate gyro drift).
- In updating step we assume accelerometer based angle estimation is close to true value .
- We take the measured angle (from the accelerometers) and the predicted angle (from the rate gyros), compute the difference, and add a portion of the difference to the final angle estimate.

$$\theta_{estimate} = \theta^{+}_{gyro} + L\left(\theta_{accel} - \theta^{+}_{gyro}\right)$$

$$\phi_{estimate} = \phi^{+}_{gyro} + L\left(\phi_{accel} - \phi^{+}_{gyro}\right)$$

$$\psi_{estimate} = \psi^{+}_{gyro} + L\left(\psi_{mag} - \psi^{+}_{gyro}\right)$$

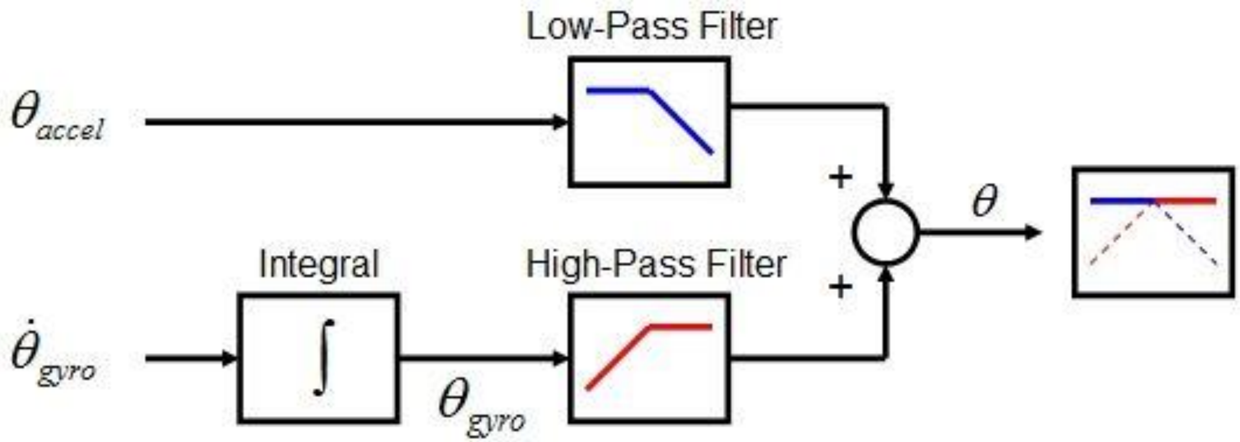
- L is numeric value between 0 and 1.

$$\begin{aligned} &if\ \ L = 0 \rightarrow \text{ gyro based angle mesurement is used} \\ &\qquad\qquad\quad \text{(accelerometer measurment become zero)} \\ &if\ \ L = 1 \rightarrow \text{ accelerometer based angle mesurement is used} \\ &\qquad\qquad\quad \text{(gyros measurment become zero)} \end{aligned}$$

- As approaches L=1, the accelerometers are trusted more and the rate gyros are trusted less. This makes the angle estimates less sensitive to rate gyro noise and biases, but more sensitive to vibration and other external forces.
- As approaches L=0, rate gyros are trusted more and the accelerometers are trusted less. This makes the angle estimates more sensitive to rate gyro noise and biases, but less sensitive to vibration and other external forces.

- This formulation can be called a "fixed-gain observer" and it looks similar to that of Kalman Filter.
- The main difference is that in a Kalman Filter, the observer gain is selected optimally using known characteristics of the physical system.
- In addition, a Kalman Filter can exploit knowledge of the physical system so that accelerometer data (and other data) needn't be converted to angles before using it to make corrections to the angle estimates.

COMPLEMENTARY FILTER



$$angle = 0.98*(angle + gyrodata*dt) + 0.02*(accData);$$

```
// Turning around the X axis results in a vector on the Y-axis
pitchAcc = atan2f((float)accData[1], (float)accData[2]) * 180 / M_PI;
*pitch = *pitch * 0.98 + pitchAcc * 0.02;

// Turning around the Y axis results in a vector on the X-axis
rollAcc = atan2f((float)accData[0], (float)accData[2]) * 180 / M_PI;
*roll = *roll * 0.98 + rollAcc * 0.02;
```

COMBINING BAROMETER 1 & 2 USING COMPLEMENTARY FILTER

Barometer 1

Barometer 2

$$H_{altitude} = H^{+}_{Barometer\,2} + L\left(H_{Barometer\,1} - H^{+}_{Barometer\,2}\right)$$

Combining Barometer Altitude and Sonar Using Complementary Filter

Barometer

Ultrasonic Sensor

$$H_{altitude} = H^{+}_{Ultrasonic\ sensor} + L\left(H_{Barometer} - H^{+}_{Ultrasonic\ sensor}\right)$$

COMBINING BAROMETER ALTITUDE INFRARED SENSOR USING COMPLEMENTARY FILTER

Barometer

Infrared Proximity sensor

$$H_{altitude} = H^{+}_{Infrared\ \text{Proximity}\ sensor} + L\left(H_{Barometer} - H^{+}_{Infrared\ \text{Proximity}\ sensor}\right)$$

Combining GPS Altitude and Barometer Altitude using Complementary Filter

GPS

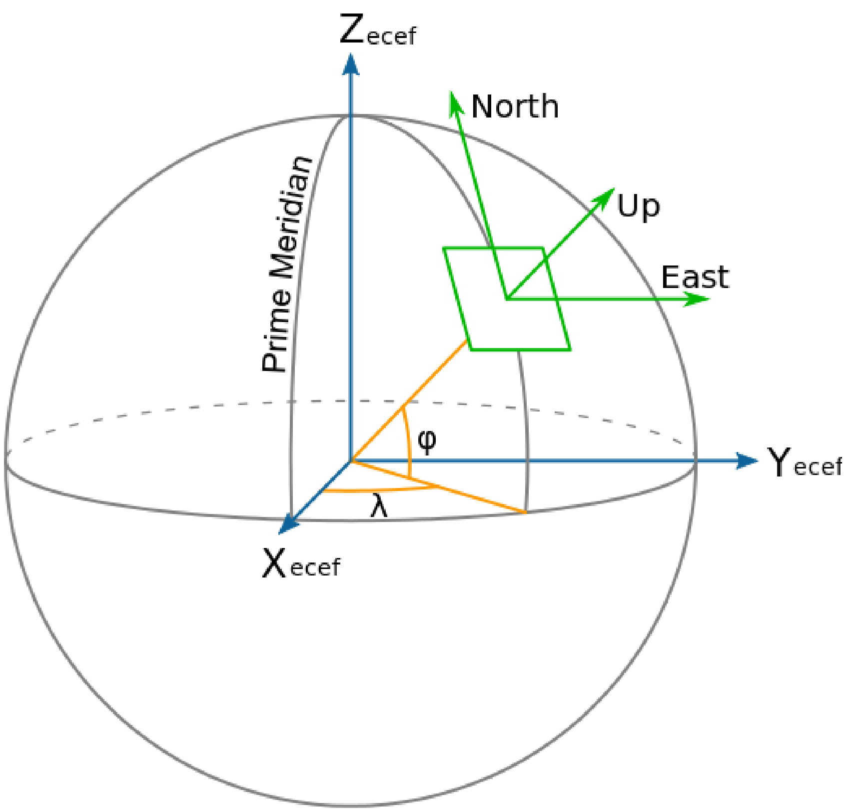
Barometer

$$H_{altitude} = H^{+}_{Barosensor} + L\left(H_{Gpsreceiver} - H^{+}_{Barometer}\right)$$

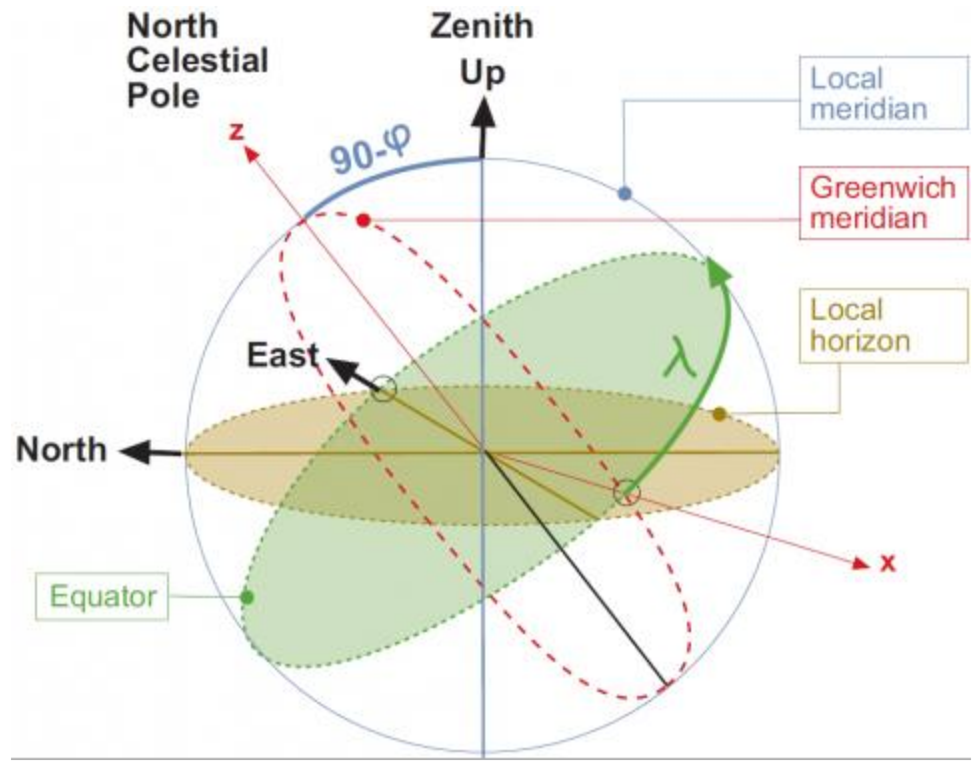
- A prime meridian is a meridian, i.e., a line of longitude, at which longitude is defined to be 0°.
- This great circle divides the sphere, e.g., the Earth, into two hemispheres. If one uses directions of East and West from a defined prime meridian, then they can be called Eastern Hemisphere and Western Hemisphere.
- Prime meridian connects pole to poles covers a distance of about 20,000 km.
- Why does the Prime Meridian (Zero Longitude) pass through Greenwich?

  -The International Meridian Conference took place in October 1884 in Washington DC. Twenty-five nations were represented at the conference by 41 delegates. The Greenwich Meridian was chosen to become the Prime Meridian of the World. There were several reasons for this; the main one being that nearly two thirds of the World's ships were already using charts based on it.

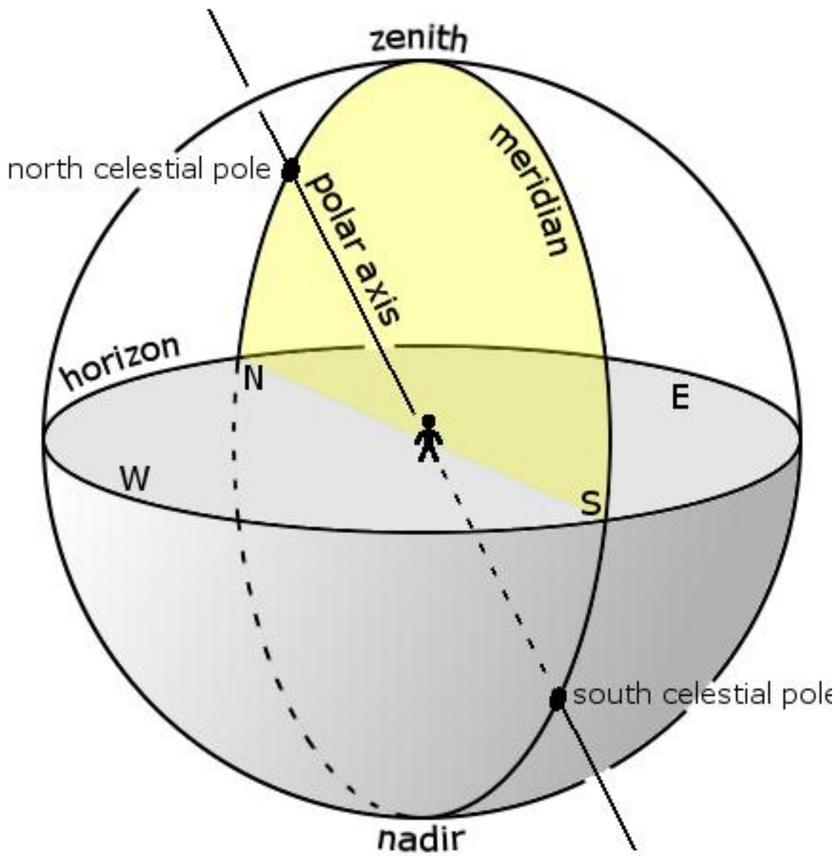
- Latitude tells you how far north or south of the Equator you are located.
- Longitude is the location of a place east or west of a north-south line called the prime meridian.
- Longitude is measured in angles ranging from 0° at the Prime Meridian to 180° at the International Date Line.
- Local meridian is a great circle passing through celestial poles and zenith of a particular location.

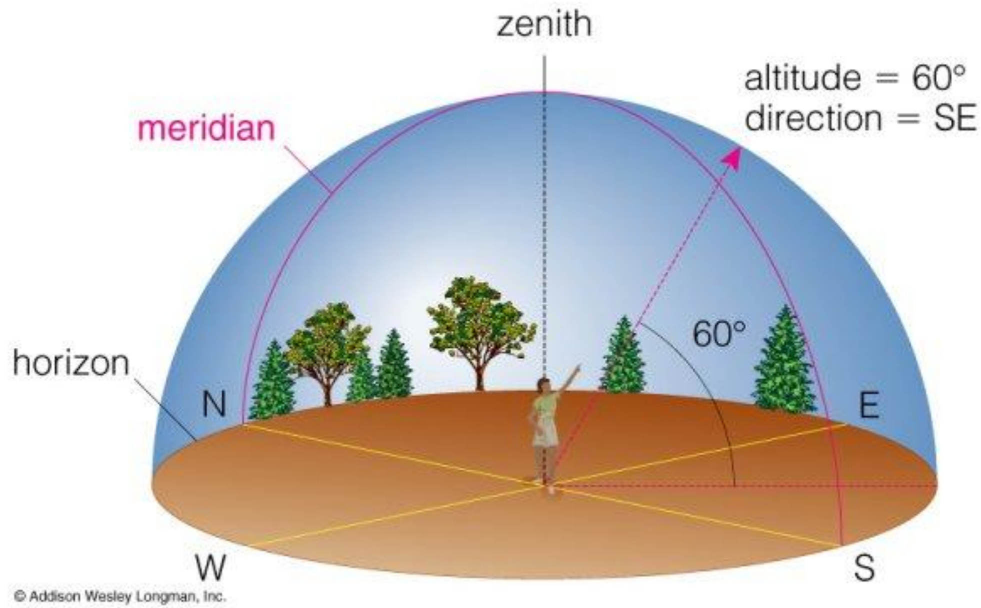


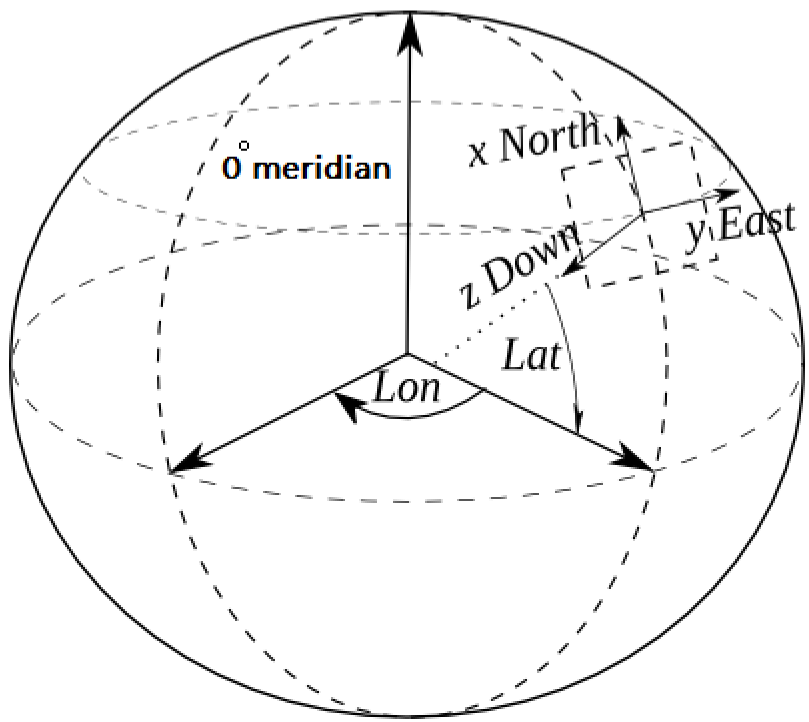
Zecef
North
Up
East
Prime Meridian
φ
λ
Yecef
Xecef



North
Celestial
Pole
Zenith
Up
Local
meridian
z
90-φ
Greenwich
meridian
Local
horizon
East
λ
North
x
Equator


zenith
north celestial pole
polar axis
meridian
horizon
N
E
W
S
south celestial pole
nadir


zenith
altitude = 60°
direction = SE
meridian
60°
horizon
N
E
W
S
© Addison Wesley Longman, Inc.


0° meridian
x North
y East
z Down
Lat
Lon

$$A \ clock-wise \ rotation \ over \ east-axis \ by \ an \ angle \ 90-\varphi \ to \ align \ the \ up-axis \ with \ the \ z-axis$$

$$R_1\left[-\left(\frac{\pi}{2}-\varphi\right)\right]$$

$$A \ clock-wise \ rotation \ over \ z-axis \ by \ an \ angle \ 90+\lambda \ to \ align \ the \ east-axis \ with \ the \ x-axis$$

$$R_3\left[-\left(\frac{\pi}{2}+\lambda\right)\right]$$

$$From \ NED \ Frame \ \text{to} \ ECEF$$

$$\begin{bmatrix} x_{ECEF} \\ y_{ECEF} \\ z_{ECEF} \end{bmatrix} = \begin{bmatrix} -\sin\lambda & -\cos\lambda\sin\varphi & \cos\lambda\cos\varphi \\ \cos\lambda & -\sin\lambda\sin\varphi & \sin\lambda\cos\varphi \\ 0 & \cos\varphi & \sin\varphi \end{bmatrix} \begin{bmatrix} x_k \\ y_k \\ z_k \end{bmatrix}_{NED}$$

$$x_{ECEF} = (-\sin\lambda, \cos\lambda, 0)$$
$$y_{ECEF} = \left(-\cos\lambda\sin\varphi, -\sin\lambda\sin\varphi, \cos\varphi\right)$$
$$z_{ECEF} = \left(\cos\lambda\cos\varphi, \sin\lambda\cos\varphi, \sin\lambda\cos\varphi\right)$$

$$From\ ECEF\ to\ NED\ Frame\ R_i(\alpha)\ i.e\ R_i^{-1}(\alpha) = R_i\left(-\alpha\right) = R_i^{T}(\alpha)$$

$$\begin{bmatrix} x_k \\ y_k \\ z_k \end{bmatrix}_{NED} = R1\left[\left(\frac{\pi}{2}-\varphi\right)\right] * R_3\left[\left(\frac{\pi}{2}+\lambda\right)\right]\begin{bmatrix} x_{ECEF} \\ y_{ECEF} \\ z_{ECEF} \end{bmatrix}$$

$$\begin{bmatrix} x_k \\ y_k \\ z_k \end{bmatrix}_{NED} = \begin{bmatrix} -\sin\lambda & \cos\lambda & 0 \\ -\cos\lambda\sin\varphi & -\sin\lambda\sin\varphi & \cos\varphi \\ \cos\lambda\cos\varphi & \sin\lambda\cos\varphi & \sin\varphi \end{bmatrix}\begin{bmatrix} x_{ECEF} \\ y_{ECEF} \\ z_{ECEF} \end{bmatrix}$$

$$x_k = \left(-\sin\lambda, -\cos\lambda\sin\varphi, \cos\lambda\cos\varphi\right)$$
$$y_k = \left(\cos\lambda, -\sin\lambda\sin\varphi, \sin\lambda\cos\varphi\right)$$
$$z_k = \left(0, \cos\varphi, \sin\varphi\right)$$

$Coordinate\ conversion\ from\ geodetic\ lat, lon\ \&\ height\ to\ ECEF\ frame$

$$x_{ECEF,k} = \left(\frac{a}{\chi_k} + h_k\right)\cos\left(Lat_k\right)\cos\left(Lon_k\right)$$

$$y_{ECEF,k} = \left(\frac{a}{\chi_k} + h_k\right)\cos\left(Lat_k\right)\sin\left(Lon_k\right)$$

$$z_{ECEF,k} = \left(\frac{a\left(1-e^2\right)}{\chi_k} + h_k\right)\cos\left(Lat_k\right)\cos\left(Lon_k\right)$$

$a \rightarrow earth's\ semi-major\ axis\ \text{in}\ meters\ (a = 6378137m)$

$e^2 \rightarrow square\ of\ the\ first\ numerical\ eccentricity\ of\ ellipsoid\ (e^2 = 6.69437999014 * 10^{-3})$

$h_k \rightarrow altitude\ of\ the\ vehicle$

$rf \rightarrow reciprocal\ flatting\ (rf = 0.0033528)$

$$\chi_k = \sqrt{1-\left(2*rf - rf^2\right)*\sin^2\left(Lat_k\right)}$$

- The Earth Centered Earth Fixed (ECEF) frame is converted into NED frame by using the rotational matrix.

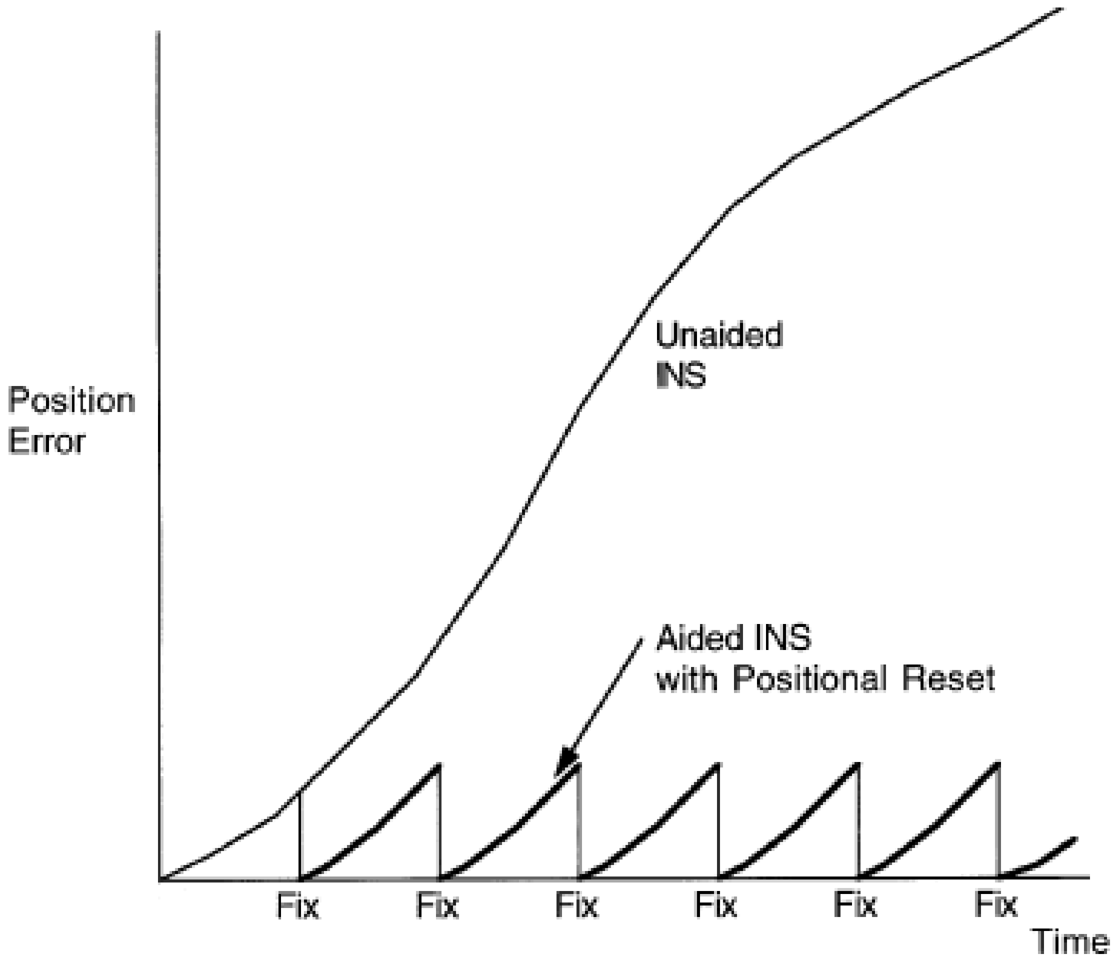
$$\begin{bmatrix} x_k \\ y_k \\ z_k \end{bmatrix}_{NED} = \begin{bmatrix} -\sin Lon_k & \cos Lon_k & 0 \\ -\sin Lat_k \cos Lon_k & -\sin Lat_k \sin Lon_k & \cos Lat_k \\ \cos Lat_k \cos Lon_k & \cos Lat_k \, s\, in Lon_k & \sin Lat_k \end{bmatrix} \begin{bmatrix} x_{ECEF,k} - xref_{ECEF} \\ y_{ECEF,k} - yref_{ECEF} \\ z_{ECEF,k} - zref_{ECEF} \end{bmatrix}$$

Combining INS Position with GPS Position using Complementary Filter

GPS

INS

Inertial Sensor



Aided INS with simple positional reset.

THANK YOU